PARAMETERIZATION OF ATMOSPHERIC HUMIDITY USING CLOUDINESS AND TEMPERATURE

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ABSTRACT

The mean relative humidity in the troposphere is expressed as a linear function of the total cloudiness, and the specific humidity as a function of temperature and cloudiness. A formula for the total precipitable water as a function of the cloudiness, the surface temperature, and the 500-mb. temperature is given. Computations of the total precipitable water over the Northern Hemisphere with the derived formula, using monthly averages, show good agreement with the estimates made by Starr, Peixoto, and Crisi.

1. INTRODUCTION

Several authors have shown that there exists a strong correlation between relative humidity and cloudiness. Among them are Telegadas and London [13] who compared the total cloud amount with the relative humidity at 850, 700, and 500 mb. Smagorinsky [11] showed that the mean relative humidity in the layers 1000–800 mb., 800–550 mb., and 550–300 mb. are linear functions of the amount of the nonconvective clouds contained in the layers.

Recently McClain [9] used satellite data to relate the cloud cover to the mean relative humidity in the layer 1000–500 mb. He showed that the positive correlation between cloudiness and relative humidity is greatly improved when a distinction is made between deep and shallow cloud conditions.

If relative humidity can be expressed as a function of cloudiness, then specific humidity becomes a function of cloudiness and temperature alone. Therefore, it is possible to compute precipitable water and make water budget studies in terms of these two variables.

Since cloudiness is a satellite-measured parameter and temperature can possibly be computed from satellite data [2], this approach will find applications in computations of mean moisture conditions from satellite data. Furthermore, both cloudiness and temperature are variables that enter into a long-range numerical prediction model proposed by the author [3]. Therefore the scheme discussed here will also find direct application in attempts at a more precise incorporation of the water budget in such a model, and in numerical studies of the general circulation of the atmosphere.

2. FORMULAS FOR SATURATION VAPOR PRESSURE AND FOR RELATIVE AND SPECIFIC HUMIDITIES

The specific humidity and the relative humidity are defined by

$$q=\frac{M_v}{M_A+M_v} \tag{1}$$

$$f=\frac{e_1}{e_s} \tag{2}$$

where q is the specific humidity, f the relative humidity, M_v the mass of water vapor, M_A the mass of dry air, e_1 the vapor pressure in the humid air, and e_s the saturation vapor pressure corresponding to the temperature of the mixture.

For ordinary values of the vapor pressure we can write the specific humidity ([5], p. 160), with good degree of approximation as

$$q=0.622\,\frac{e_1}{p^*} \tag{3}$$

where p^* is the pressure of the humid air.

From equation (2) and the equation of a perfect gas, formula (3) becomes

$$q=\frac{0.622}{R\rho^*}\left(\frac{e_s}{T^*}\right)f \tag{4}$$

where R, ρ^*, and T^* are the gas constant, the density, and the absolute temperature, respectively, of the humid air.

The saturation vapor pressure of water is a function of temperature and can be expressed with a simple formula [7]. For the liquid phase in the range from $-40°$ to $40°$ C., it can be represented by

$$e_s=a_1+b_1t^*+c_1t^{*2}+d_1t^{*3}+l_1t^{*4} \tag{5}$$

where e_s is the saturation vapor pressure in millibars and where $t^*=T^*-273.16°$ C. Using the values given by Byers ([5], p. 158), we obtain the following values for the coefficients: $a_1=6.115$, $b_1=0.42915$, $c_1=0.014206$, $d_1=3.046\times10^{-4}$, and $l_1=3.2\times10^{-6}$. Comparison of the values of e_s obtained from equation (5) using these coefficients, with the corresponding values from Byers' table, shows excellent agreement in the whole range from −40° to 40° C.

Next we will show that the mean relative humidity over periods of a month or a season, $\bar{f}$, can approximately be written as

$$\bar{f}=\bar{f}_m+f_1(z) \tag{6}$$

where

$$\bar{f}_m=\frac{1}{H_T}\int_0^{H_T}\bar{f}dz$$

and H_T is the height of the troposphere.

Figure 1 shows $\bar{f}-\bar{f}_m$ for summer computed from London's relative humidity values ([8], p. 92), as a function of height, for different latitudes. From these graphs and similar ones for the other seasons we can compute the mean value of $f_1(z)$ which can be taken as the same for all latitudes. $f_1(z)$ can be represented by

$$f_1(z)=A_0+A_1z+A_2z^2 \tag{7}$$

If $f_1(z)$ is in percent and z in kilometers, the values of A_0, A_1, and A_2 are those in table 1.

TABLE 1.—*Seasonal values of* A_0, A_1, *and* A_2

Season	A_0	A_1	A_2
Winter	22.45	−9.00	0.62
October	20.16	−7.54	.50
Summer	23.50	−8.36	.52
April	20.20	−8.00	.55

When $z=z_1$, formula (6) becomes

$$(\bar{f})_{z=z_1}=\bar{f}_m+f_1(z_1) \tag{8}$$

Subtracting (8) from (6) we obtain

$$\bar{f}=(\bar{f})_{z=z_1}+f_1(z)-f_1(z_1)$$

Therefore the three-dimensional field of relative humidity can be computed from the values at any given height.

Looking for a linear relation between mean relative humidities ($\bar{f}_m$) and total cloud amounts ($\bar{\epsilon}$), we found that the best fit is obtained when the mean relative humidity

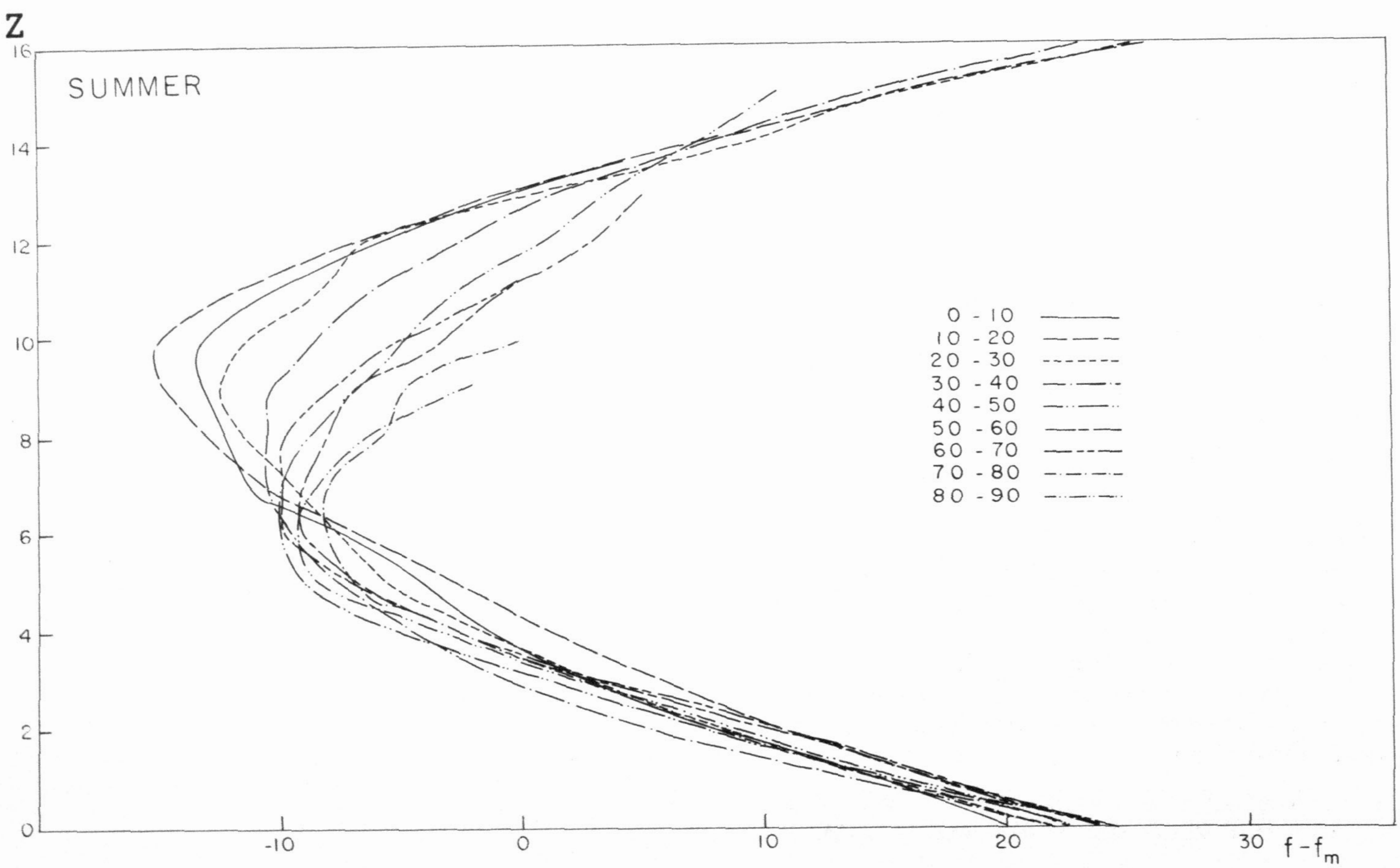

FIGURE 1.—The abscissa is the relative humidity minus the mean tropospheric relative humidity for summer in percent and the ordinate is the height in km. Curves for mean values over zonally averaged 10° latitude regions are shown.

is averaged over the whole troposphere. Figure 2 shows the results of the computations in which only London's [8] zonally averaged values have been used. The dots are the average values for 10° latitude belts and the lines are fitted by eye.

The formula corresponding to these lines is

$$\bar{f}_m = B_1 + B_2\bar{\epsilon} \tag{9}$$

If $\bar{f}_m$ and $\bar{\epsilon}$ are in percent, then the values of B_2 are found to be the same for all seasons, and are equal to 0.5, and B_1 is equal to 24.0 in autumn and summer, 21.0 in spring, and 25.5 in winter. Except in winter, equation (9) is not valid in lower latitudes; i.e., below 20° in spring and summer and below 10° in autumn. In winter it is not valid in higher latitudes; i.e., above 70°.

The approximation given by (9) can be judged by inspection of figure 2, and no claim of great accuracy is made here. More exact studies such as those of Smagorinsky [11] and McClain [9] and a more complete set of data are needed to determine a better relationship between cloudiness and relative humidity.

Substituting (9) in (6) we obtain

$$\bar{f} = B_1 + B_2\bar{\epsilon} + f_1(z) \tag{10}$$

This formula shows that the relative humidity at any level is proportional to the total cloud cover.

3. TOTAL PRECIPITABLE WATER

Multiplying (4) by ρ^* we obtain the water vapor per unit volume

$$\rho^* q = \frac{0.622}{R} \frac{e_s}{T^*} f \tag{11}$$

We shall assume that

$$\overline{(e_s/T^*)f} = (\bar{e}_s/\bar{T}^*)\bar{f},$$

where a bar denotes an average over the considered time interval and $\bar{e}_s(T^*) = e_s(\bar{T}^*)$. Therefore

$$\overline{\rho^* q} = \frac{0.622}{R} \frac{e_s(\bar{T}^*)}{\bar{T}^*} \bar{f} \tag{12}$$

Substituting (10) in (12), we obtain

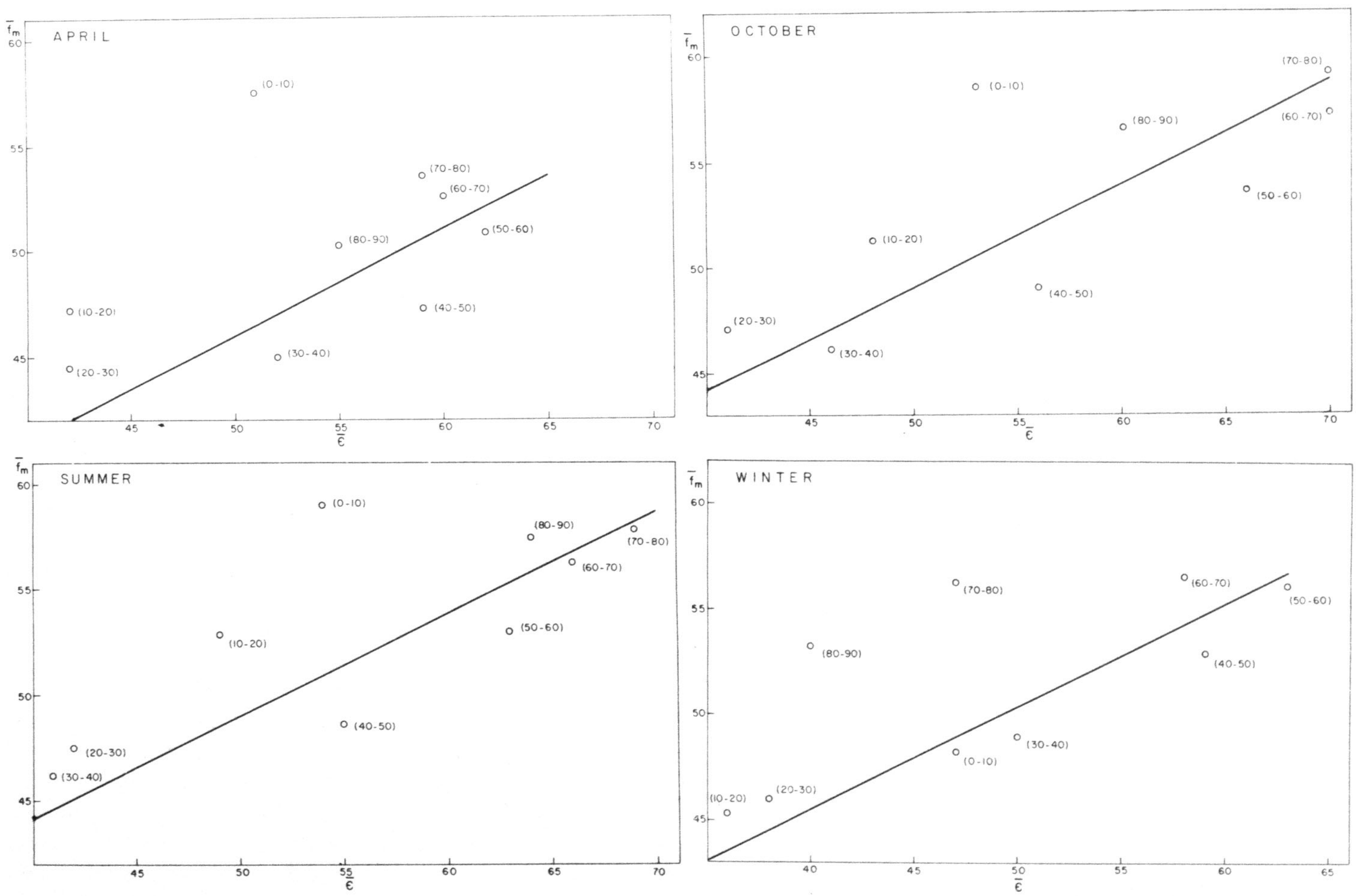

FIGURE 2.—Mean relative humidity in the troposphere versus total cloud amount. The dots are the average values for 10° latitude belts. The continuous line corresponds to equation (9).

$$\overline{\rho^* q}=\frac{0.622}{R}P_1[B_1+B_2\bar{\epsilon}+f_1(z)] \tag{13}$$

where $P_1=e_s(\bar{T}^*)/\bar{T}^*$ is a function of the temperature only.

We assume that

$$\bar{T}^*=\bar{\beta}(H-z)+\bar{T} \tag{14}$$

where $\bar{T}$ is the time-averaged temperature at the height H, z is the vertical coordinate measured from sea level, and $\bar{\beta}$ is the time-averaged lapse rate which will be taken as a function of the horizontal coordinates and time.

The total precipitable water in the layer of height H is given by

$$\bar{w}=\int_h^H \overline{q\rho^*}dz \tag{15}$$

where h is the height of the earth's surface. Substituting (13) in (15) we obtain

$$\bar{w}=\frac{0.622}{R}\int_h^H P_1[B_1+B_2\bar{\epsilon}+f_1(z)]dz \tag{16}$$

Substituting (14) in (16) and replacing z by $\bar{T}^*$ as the variable in the integrand, we obtain

$$\bar{w}=-\frac{0.622}{R\bar{\beta}}\int_{\bar{T}_B}^{\bar{T}}\left[B_1+B_2\bar{\epsilon}+f_1\left(\frac{\bar{T}-\bar{T}^*}{\bar{\beta}}+H\right)\right]P_1(\bar{T}^*)d\bar{T}^* \tag{17}$$

where $\bar{T}_B$ is the time-averaged surface air temperature.

Integrating (17) we obtain

$$\bar{w}=\frac{0.622}{R}\left[(B_1+B_2\bar{\epsilon})\frac{L_3}{B_2}+\frac{A_0L_3}{B_2}+\frac{A_1I_6H}{2}\right]+\frac{0.622}{R}\frac{A_2}{\bar{\beta}^3}\left[L_q-2\bar{T}_BL_8+\frac{\bar{\beta}\bar{T}_B^2}{B_2}L_3\right] \tag{18}$$

where

$$L_3=\frac{B_2}{\bar{\beta}}\left\{a\ln\frac{\bar{T}_B}{\bar{T}_B-N}+bN-\frac{cN^2}{2}+\frac{dN^3}{3}-\frac{lN^4}{4}+(cN-dN^2+lN^3)\bar{T}_B+\left(dN-\frac{3lN^2}{2}\right)\bar{T}_B^2+Nl\bar{T}_B^3\right\},$$

$$I_6=\frac{2\bar{T}_B}{B_2N}L_3-\frac{2}{\bar{\beta}N}L_8,$$

$$L_8=aN-\frac{bN^2}{2}+\frac{cN^3}{3}-\frac{dN^4}{4}+\frac{lN^5}{5}+(bN-cN^2+dN^3-lN^4)\bar{T}_B+\left(cN-\frac{3}{2}dN^2+2lN^3\right)\bar{T}_B^2+(dN-2lN^2)\bar{T}_B^3+lN\bar{T}_B^4$$

and

$$L_q=-\frac{aN^2}{2}+\frac{bN^3}{3}-\frac{cN^4}{4}+\frac{dN^5}{5}-\frac{lN^6}{6}+(aN-bN^2+cN^3-dN^4+lN^5)\bar{T}_B+\left(bN-\frac{3cN^2}{2}+2dN^3-\frac{5lN^4}{2}\right)\bar{T}_B^2+\left(cN-2dN^2+\frac{10lN^3}{3}\right)\bar{T}_B^3+\left(dN-\frac{5lN^2}{2}\right)\bar{T}_B^4+lN\bar{T}_B^5$$

where $N=\bar{\beta}(H-h)$ and a, b, c, d, and l are the corresponding coefficients obtained in formula (5) when we expand the powers of $t^*=T^*-273.13$°C. and add terms containing the same power of T^*.

We shall take a generalized lapse rate given by

$$\bar{\beta}=\frac{\bar{T}_B-\bar{T}}{H-h}$$

where $\bar{T}_B$ is the observed surface temperature, h is the surface topography, and $\bar{T}$ the temperature at the height H.

When using formula (5) to compute e_s, we are assuming that the water contained in the layer of height H is supercooled when the temperature is below freezing. This condition is satisfied if we take H sufficiently small that the temperatures are above say −20° C. Since the total amount of water contained above 500 mb. is negligible compared with the amount below that surface [12], we will take H=5.5 km. Therefore, except at higher latitudes, the assumption that there is supercooled water in the clouds is probably satisfied.

The best estimates of the distribution of precipitable water in the Northern Hemisphere published to date are probably those of Peixoto and Crisi [10] and Starr, Peixoto, and Crisi [12] from IGY data. Their results are averages for the year 1958, and for the 6-month periods October to March (winter) and April to September (summer) of the same year.

In order to be able to compare the results of the above formula with the data presented by Starr, Peixoto, and Crisi, computations for the same periods used by them were carried out, except that only the average of the computed values for November and February are used as representative of the period from October to March, and the average of the computed values for May and August as representative of the period April to September. Furthermore, we use monthly normal values.

The normal surface air temperatures used in the computations were based largely on recent WMO normals [14]. The normal 500-mb. temperatures were computed from upper-air height data prepared by Hennig [6]; the normal cloud cover was obtained from London [8]; and the mean surface topography from Berkofsky and Bertoni [4]. The computations were carried out in the same region and

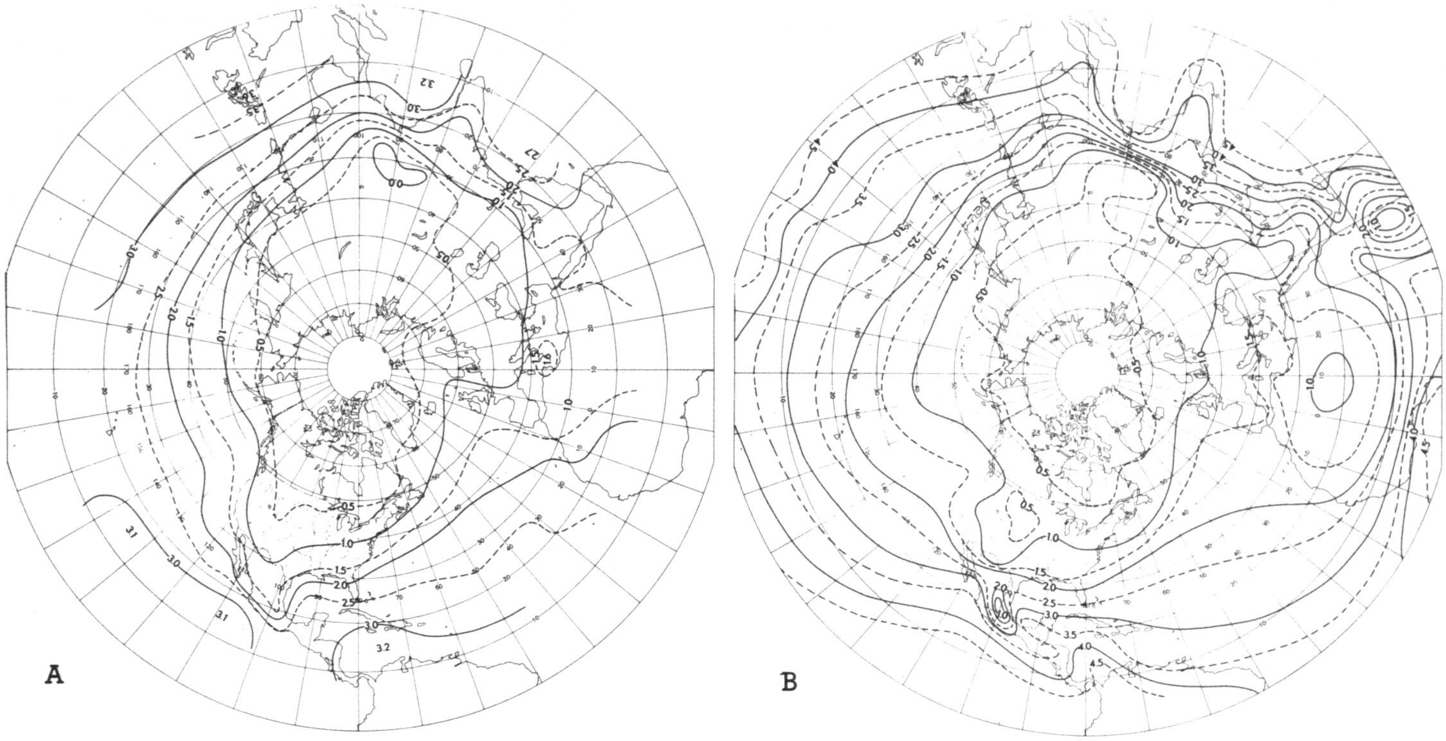

FIGURE 3.—Total precipitable water in the troposphere for winter, in gm. cm.$^{-2}$: (A) using formula (18), with monthly normal cloud cover values and monthly normal temperatures, and (B) the corresponding values for 1958 obtained by Starr, Peixoto, and Crisi.

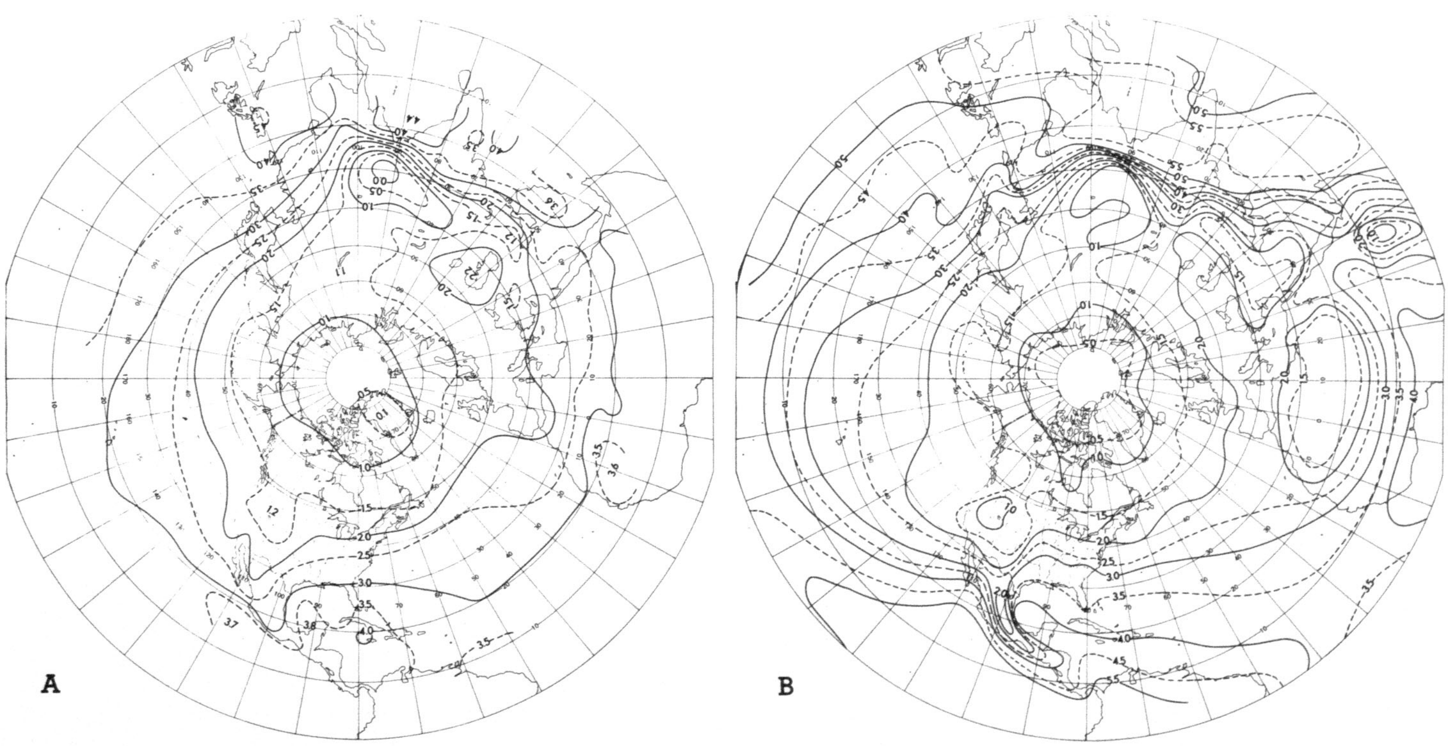

FIGURE 4.—Total precipitable water in the troposphere for summer, in gm. cm.$^{-2}$: (A) using formula (18), with monthly normal cloud cover values and monthly normal temperatures, and (B) the corresponding values for 1958 obtained by Starr, Peixoto, and Crisi.

with the same grid points as in the author's previous work ([1], p. 94).

Figures 3A and 4A are the results of the computations for winter and summer respectively; and figures 3B and 4B the corresponding values published by Starr, Peixoto, and Crisi. Their comparison shows a general good agreement. Some of the discrepancies are due to the fact that their values are for the year 1958, while ours correspond to a climatological normal. Therefore, a detailed comparison is probably not warranted. However, it is clear from the comparison that the essential features in the general pattern, as well as the magnitude of the values, are in agreement.

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